3dVar

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The analysis can be obtained by numerically minimizing a cost function $J$. This cost function and its gradient are given by

$$J(x) = \frac{1}{2}[(x - x^b)^T B^{-1}(x - x^b) + (y - Hx)^T R^{-1}(y - Hx)]$$

$$\nabla J(x) = B^{-1}(x - x^b) - H^T R^{-1}(y - Hx),$$

where $B, R$ are the background and observation error covariance matrices, $H$ is the observation operator.

The method to obtain the analysis by minimizing a cost function is called 3dVar.

Setting $\nabla J$ to 0 leads to the analysis update equation:

$$x^a = x^b + K(y - Hx^b)$$

$$K = BH^T (HBH^T + R)^{-1}$$

$K$ is called the Kalman gain matrix. This equation can also be obtained by using Bayes’ Theorem and assuming gaussian pdf’s or by a least square estimation.
the solution is obtained by minimizing the cost function numerically using e.g. the conjugate gradient method

- advantages of 3dVar:
  - variational scheme, allows e.g. outer loops (necessary to deal with nonlinearities), varQC of observations
  - radiances can be assimilated directly (no 1dVar needed)
  - no adjoint model as in 4dVar required
  - solves global problem, no localization issues

- disadvantages of 3dVar:
  - the $B$ matrix is constant in time.
  - comparison between observations and model equivalents not at appropriate time (different in fgat-3dVar); no 4D method
  - adjoint of observation operator needed

- a method to speed up convergence is the preconditioning; this will be discussed in the context of the hybrid method.